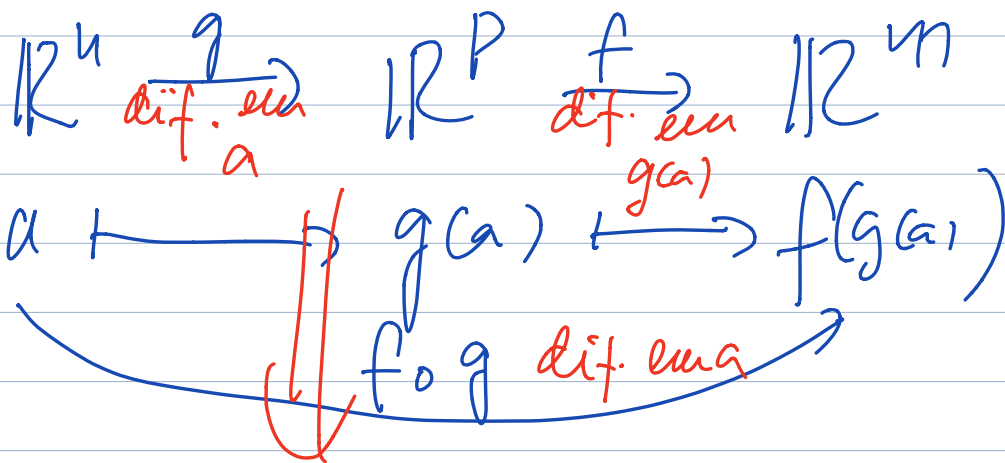


CDI-II - Prática F.4 24/3/21

Ficha 4

Derivadas da função composta



$$D(f \circ g)(a) = Df(g(a)) Dg(a)$$

$m \times n \qquad m \times p \qquad p \times n$

1 - trivial : $g(1,1) = (1,0)$

$$D(f \circ g)(1,1) = Df(1,0) Dg(1,1)$$

$$\boxed{(1,1)} \longrightarrow g(1,1) \longrightarrow f(g(1,1))$$

$$\quad \quad \quad \downarrow \text{||}$$

$$\quad \quad \quad \boxed{(1,0)}$$

$$\text{-----} \underset{\quad}{\llcorner} \text{-----}$$

2 - $\gamma \equiv$ "gamma", $\sigma \equiv$ "sigma"

$$v(t) = F(\gamma(t))$$

$$\mathbb{R} \xrightarrow{\gamma} \mathbb{R}^3 \xrightarrow{F} \mathbb{R}$$

$$t \mapsto \gamma(t) \mapsto F(\gamma(t))$$

$$v'(t) = DF(\gamma(t)) \gamma'(t)$$

$$1 \times 1$$

$$1 \times 3$$

$$3 \times 1$$

$$\sigma'(t) = \begin{bmatrix} 2\dot{x}t & 2t^2 & 2\dot{y}t \end{bmatrix} \begin{bmatrix} \dot{x}t \\ 2t \\ -\dot{x}t \end{bmatrix} = 4t^3$$

\uparrow
 $DF(\gamma(t))$

$$DF(x, y, z) = \begin{bmatrix} 2x & 2y & 2z \end{bmatrix}$$

$$DF(\gamma(t)) = \begin{bmatrix} 2\dot{x}t & 2t^2 & 2\dot{y}t \end{bmatrix}.$$

$$\gamma(t) = (\dot{x}t, t^2, \dot{y}t)$$

$\uparrow \quad \uparrow \quad \uparrow$
 $x(t) \quad y(t) \quad z(t)$

$$\gamma'(t) = \begin{bmatrix} \dot{y}t \\ 2t \\ -\dot{x}t \end{bmatrix}$$

$$\sigma(t) = 4t^3.$$

Alternativamente, usar a regra de cadeia.

$$\sigma(t) = F(x(t)) = F(x(t), y(t), z(t))$$

$$\sigma'(t) = \frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt} + \frac{\partial F}{\partial z} \frac{dz}{dt}$$

$$x(t) = 2e^{2t}$$

$$y(t) = t^2$$

$$z(t) = \cos t$$

$$(x(t), y(t), z(t))$$

$$(t)$$

$$= \cancel{2e^{2t} \cos t} + 2t \times 2t + \cancel{2 \cos t (-2e^{2t})}$$

$$= 4t^2$$

$$3 - \mathbb{R}^2 \xrightarrow{g} \mathbb{R}^3 \xrightarrow{f} \mathbb{R}$$

$$(0,0) \mapsto \underset{(0,1,2)}{g(0,0)} \mapsto f(g(0,0)) = f(0,1,2)$$

$$D(f \circ g)(0,0) = Df(\overset{g(0,0)}{(0,1,2)}) Dg(0,0)$$

$$= \begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

... 0

$$D(f \circ g)(0,0) = D(f \circ g)(0,0) \vee$$

$$\begin{bmatrix} \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

etc.

Alternativamente, usar a regra de cadeia.

$$(f \circ g)(x, y) = f(g(x, y))$$

$$h(x, y) = f(u(x, y), v(x, y), w(x, y))$$

$$f(u, v, w) = v e^u + u w^2$$

$$\frac{\partial h}{\partial x}(x, y) = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial f}{\partial w} \frac{\partial w}{\partial x}$$

etc.

$$\begin{array}{cc} (0, 1, 2) & (0, 0) \\ (u, v, w) & (x, y) \\ \parallel & \\ g(x, y) & \end{array}$$

$$4- \sigma(x) = f(g(x))$$

$$g(x) = (\sin x, x + e^x)$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$g: \mathbb{R} \rightarrow \mathbb{R}^2$$

$$\mathbb{R} \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^3$$

$$\begin{array}{ccc} x \mapsto & g(x) = & \mapsto f(g(x)) \\ & (\sin x, x + e^x) & \\ \text{---} & \searrow & \nearrow \\ & \sigma(x) = (f \circ g)(x) & \end{array}$$

$$\sigma'(x) = Df(g(x)) Dg(x)$$

$$\sigma'(0) = Df(g(0)) Dg(0)$$

$$\sigma'(0) = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} \text{ etc.}$$

————— \cup —————

$$5-a) \mathbb{R}^3 \xrightarrow{f} \mathbb{R}^3 \xrightarrow{g} \mathbb{R}$$

————— \cup —————

$g \circ f$

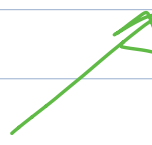
$$\begin{aligned} (x, y, z) &\mapsto f(x, y, z) \rightarrow g(f(x, y, z)) \\ (1, 1, 0) &\mapsto (2, 2, 1) \mapsto g(2, 2, 1) \end{aligned}$$

$$D(g \circ f)(x, y, z) = Dg(f(x, y, z)) Df(x, y, z)$$

$(1, 1, 0)$ $(2, 2, 1)$ $(1, 1, 0)$

$$= \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

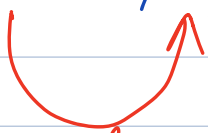
$$\frac{\partial}{\partial y} (g \circ f)(1, 1, 0)$$


 etc.

$$5-b) \frac{\partial}{\partial z} (g \circ f)(0,1,0)$$

$$h(x,y,z) = g(f(x,y,z))$$

$$h(x,y,z) = g(u(x,y,z), v(x,y,z), w(x,y,z))$$



$$\frac{\partial h}{\partial z} = \frac{\partial g}{\partial u} \left(\frac{\partial u}{\partial z} \right) + \frac{\partial g}{\partial v} \left(\frac{\partial v}{\partial z} \right) + \frac{\partial g}{\partial w} \left(\frac{\partial w}{\partial z} \right)$$

$$\begin{pmatrix} x,y,z \\ 0,1,0 \end{pmatrix}$$

$$f(x,y,z) = (u,v,w)$$

$$f(0,1,0) = (1,1,0)$$

$$(0,1,0)$$

etc.

$$f - F(x, y, g(x, y)) = 0 \quad \checkmark$$

\Leftrightarrow

$$z = g(x, y) \equiv z(x, y)$$

$$\boxed{\frac{\partial F}{\partial z}(x, y, z) \neq 0} \Rightarrow Dg(x, y)?$$

$$F(x, y, z(x, y)) = 0$$

$$\left(\frac{\partial g}{\partial x}\right) = \frac{\partial z}{\partial x}$$

$$\frac{\partial g}{\partial y} = \frac{\partial z}{\partial y}$$

$$h(x, y) = 0$$

$$0 = \frac{\partial h}{\partial x}(x, y) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} \left[\frac{\partial z}{\partial x} \right]$$

$$0 = \frac{\partial h}{\partial y}(x, y) = \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} \left[\frac{\partial z}{\partial y} \right]$$

$$\frac{\partial g}{\partial x}(x, y) = - \frac{\frac{\partial F}{\partial x}(x, y, g(x, y))}{\frac{\partial F}{\partial z}(x, y, g(x, y))}$$

$$\frac{\partial g}{\partial y}(x, y) = - \frac{\frac{\partial F}{\partial y}(x, y, g(x, y))}{\frac{\partial F}{\partial z}(x, y, g(x, y))}$$

————— \downarrow —————

$$6 - \frac{\partial}{\partial x} g \left(f(x^2, xy, x+y), 3xy, 2x+y \right)$$

$$g \left(\overbrace{f(x^2, xy, x+y)}^{u(x, y)}, \overbrace{3xy}^{v(x, y)}, \overbrace{2x+y}^{w(x, y)} \right)$$

$$= g(u(x, y), v(x, y), w(x, y))$$

$$h(x, y) = g(u(x, y), v(x, y), w(x, y))$$

$$h(x, y) = g(x^2, 3xy, x+y)$$

$$v(x, y) = 3xy$$

$$w(x, y) = 2x + y$$

$$\frac{\partial h}{\partial x} = \frac{\partial g}{\partial u} \left[\frac{\partial u}{\partial x} \right] + \frac{\partial g}{\partial v} \left(\frac{\partial v}{\partial x} \right) + \frac{\partial g}{\partial w} \left(\frac{\partial w}{\partial x} \right)$$

$$h(x, y) = g(a(x, y), b(x, y), c(x, y))$$

$$\frac{\partial h}{\partial x} = \frac{\partial g}{\partial a} \frac{\partial a}{\partial x} + \frac{\partial g}{\partial b} \frac{\partial b}{\partial x} + \frac{\partial g}{\partial c} \frac{\partial c}{\partial x}$$

$\frac{\partial g}{\partial u}$ | $\frac{\partial g}{\partial a}$ \equiv derivada parcial
de g na primeira
variável

\uparrow | \uparrow

(u, v, w) | (a, b, c)